U.G. Sem VI 02.02.2026

$\boxed{\text{MJC} - 11}$ METRIC SPACE

Definition

Let X be a non-empty set and d be a function defined on $X \times X$ into the set $\mathbb{R}$ of real numbers such that the image of any ordered pair (x, y) of $X \times X$ is denoted by $d(x, y)$. Then d is called a metric (or distance) function if and only if d satisfies the following axioms.

1. $d(x, y) \geq 0 \ \forall \ x, y \in X$

In other words, the distance between any two points is non-negative real number.

2. $d(x, y) = 0 \Leftrightarrow x = y$

i.e. distance between two points is zero iff the two points coincide.

3. $d(x, y) = d(y, x) \ \forall \ x, y \in X$

(SYMMETRIC PROPERTY)

4. $d(x, y) \leq d(x, z) + d(z, y) \ \forall x, y, z \in X$

[TRIANGULAR INEQUALITY]

The set X along with metric d is
i.e. (X, d) is called metric space.

METRIZABLE

A non-empty set X is called metrizable iff a metric d can be defined on X.

Pseudo-metric space

If $d(x, x) = 0 \; \forall \; x \in X$
i.e. $x = y \Rightarrow d(x, y) = 0$ but
$d(x, y) = 0$ does not necessarily imply that $x = y$.

And other axioms are satisfied then it is a pseudo-metric space.

Imp. Thus every metric space is a pseudo-metric space but every pseudo-metric space is not a metric space.